Tensor Galileons and Gravity

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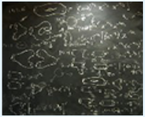

Introduction and Motivation

- In physics we encounter differential equations up to second order in derivatives
- In cosmology $\rightsquigarrow$ interest in higher derivative self-interactions, e.g. for scalar fields

Most general theory with second-order field equations?

Gravity$^+$

Most general metric theory with second-order field equations in D dimensions?

- $D = 4 \quad \rightsquigarrow \quad$ General relativity
- $D = 5 \quad \rightsquigarrow \quad$ Einstein-Hilbert + Gauss-Bonnet
- $D = D \quad \rightsquigarrow \quad$ Lovelock

Most general metric-scalar theory with second-order field equations in $D = 4$?

- Horndeski theory

Scalar$^+$

- The answer in flat spacetime of $D \geq n$ is given by (a sum over) Galileons
  Nicolis, Rattazzi, Trincherini '08

$$\mathcal{L}_{n+1}[\pi] = \mathcal{A}_{(2n)}^{i_1 \ldots i_n j_1 \ldots j_n} \, \partial_{i_1}\pi \, \partial_{j_1}\pi \, \partial_{i_2}\partial_{j_2}\pi \ldots \partial_{i_n}\partial_{j_n}\pi \ ,$$

  where

$$\mathcal{A}_{(2n)}^{i_1 \ldots i_n j_1 \ldots j_n} = \frac{1}{(D-n)!} \varepsilon^{i_1 \ldots i_n k_1 \ldots k_{D-n}} \varepsilon^{j_1 \ldots j_n}{}_{k_1 \ldots k_{D-n}} \ .$$

- The name reflects the internal Galilean invariance under $\delta\pi = c + b_i x^i$
- The first few Lagrangians are

$$\mathcal{L}_2 = -\tfrac{1}{2}(\partial\pi)^2, \quad \mathcal{L}_3 = -\tfrac{1}{2}(\partial\pi)^2\Box\pi, \quad \mathcal{L}_4 = -\tfrac{1}{2}(\partial\pi)^2\left[(\Box\pi)^2 - (\partial_i\partial_j\pi)^2\right], \ \ldots$$

- Covariantization yields scalar-tensor theories in any D (in $4 \equiv$ Horndeski)
  Deffayet, Esposito-Farese, Vikman '09

- Also, scalars with up to 2[nd] order eoms, Galileon-type p-forms, multiple species.
  Deffayet, Deser, Esposito-Farese '09, '10; Deffayet, Mukohyama, Sivanesan '16 &c.

Our Goals

- ✓ A universal, index-free formulation for all Galileons and their generalizations
  - with graded variables $\rightsquigarrow$ motivated by the "double-ϵ" structure
- ✓ Generalization to mixed-symmetry tensor fields (p, q) (beyond spin-1)
  - Young diagrams with 2 columns as generalized gauge fields Curtright '85
  - Dual graviton / exotic dualizations de Medeiros, Hull '02
  - E_{11} West '04 / Exotic branes Bergshoeff, Riccioni '10; A.Ch., Gautason, Moutsopoulos, Zagermann '13

Graded formalism

Extend the bosonic coordinates (x^i) by two sets of anticommuting (θ^i) and (χ^i):

$$\theta^i\theta^j = -\theta^j\theta^i\ , \quad \chi^i\chi^j = -\chi^j\chi^i\ , \quad \theta^i\chi^j = \chi^j\theta^i\ .$$

Represent a p-form $\omega^{(p)}$ in two ways:

$$\boldsymbol{\omega}^{(p)} = \tfrac{1}{p!}\omega_{i_1\dots i_p}\theta^{i_1}\dots\theta^{i_p}\ , \quad \widetilde{\boldsymbol{\omega}}^{(p)} = \tfrac{1}{p!}\omega_{i_1\dots i_p}\chi^{i_1}\dots\chi^{i_p}\ .$$

Introduce two nilpotent and mutually commuting exterior derivatives:

$$\mathbf{d} = \theta^i\partial_i \quad \text{and} \quad \widetilde{\mathbf{d}} = \chi^i\partial_i\ .$$

Use Berezin integration to integrate over the graded variables:

$$\int \mathrm{d}\theta\,\theta = 1\ , \quad \int d^D\theta\,\theta^{i_1}\dots\theta^{i_D} = \varepsilon^{i_1\dots i_D}\ .$$

Scalar and p-form Galileons

Scalar Galileon

$$\mathcal{L}_{n+1}[\pi] = -\frac{1}{(D-n)!}\int \mathrm{d}^D\theta\,\mathrm{d}^D\chi\,\boldsymbol{\eta}^{D-n}\,\pi\,(\mathbf{d}\tilde{\mathbf{d}}\pi)^n\,, \qquad (\boldsymbol{\eta} = \eta_{ij}\theta^i\chi^j)$$

The field equations are 2nd order: $E_{n+1} = -\frac{n+1}{(D-n)!}\int \mathrm{d}^D\theta\,\mathrm{d}^D\chi\,\boldsymbol{\eta}^{D-n}(\mathbf{d}\tilde{\mathbf{d}}\pi)^n = 0$

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p-form Galileon

$$\mathcal{L}_{2n}[\omega] = \frac{1}{(D-(p+2)n+1)!}\int \mathrm{d}^D\theta\,\mathrm{d}^D\chi\,\boldsymbol{\eta}^{D-(p+2)n+1}\,\mathbf{d}\boldsymbol{\omega}\,\tilde{\mathbf{d}}\tilde{\boldsymbol{\omega}}\,(\mathbf{d}\tilde{\mathbf{d}}\tilde{\boldsymbol{\omega}})^{n-1}\,(\tilde{\mathbf{d}}\mathbf{d}\boldsymbol{\omega})^{n-1}\,.$$

N.B.: For $p = 2k+1 \Rightarrow (\mathbf{d}\tilde{\mathbf{d}}\boldsymbol{\omega})^2 = (\mathbf{d}\tilde{\mathbf{d}}\tilde{\boldsymbol{\omega}})^2 = 0 \ \rightsquigarrow$ only $n = 1$ for odd-forms, e.g.:

$$\mathcal{L}_{\text{Maxwell}}[A] = -\frac{1}{2}\int \mathrm{d}^4\theta\,\mathrm{d}^4\chi\,\boldsymbol{\eta}^2\,\mathbf{d}\boldsymbol{A}\,\tilde{\mathbf{d}}\tilde{\boldsymbol{A}}\,,$$

unless mixed contractions are considered for $p = 3, 5, \ldots$ Deffayet et al. '16

Mixed-symmetry tensor fields

$\omega_{[i_1\ldots i_p][j_1\ldots j_q]} \rightsquigarrow (p,q)$ tensor field

$GL(D)$-irreducibility

$$T_{[i_1\ldots i_p j_1]\ldots j_q} = 0 \quad \text{and} \quad T_{[i_1\ldots i_p][j_1\ldots j_q]} = T_{[j_1\ldots j_q][i_1\ldots i_p]}\ , \ \text{for } p = q\ .$$

e.g. for $p+q=2$, a 2-form (2,0) and a graviton (1,1);
for $p+q=3$, a 3-form (3,0) and a mixed (2,1);
for $p+q=4$, a 4-form (4,0), a mixed (3,1) and a "special" mixed (2,2); etc.

Natural description in terms of the graded variables:

$$\boldsymbol{\omega}^{(p,q)} = \tfrac{1}{p!q!}\omega_{i_1\ldots i_p j_1\ldots j_q}\theta^{i_1}\ldots\theta^{i_p}\chi^{j_1}\ldots\chi^{j_q}\ ,$$
$$\widetilde{\boldsymbol{\omega}}^{(q,p)} = \tfrac{1}{p!q!}\omega_{i_1\ldots i_p j_1\ldots j_q}\chi^{i_1}\ldots\chi^{i_p}\theta^{j_1}\ldots\theta^{j_q}\ ,$$

and the same derivatives $\mathbf{d}$ and $\widetilde{\mathbf{d}}$; no need for additional ingredients.

Mixed-symmetry Galileon and its Symmetry

For a single mixed-symmetry tensor field ω, the Galileon is ($k = (p+q+2)n - 1$):

$$S_{2n}[\omega] = \frac{1}{(D-k)!} \int \mathrm{d}^D x \int \mathrm{d}^D\theta \, \mathrm{d}^D\chi \, \boldsymbol{\eta}^{D-k} \mathbf{d}\boldsymbol{\omega} \, \tilde{\mathbf{d}}\tilde{\boldsymbol{\omega}} \, (\mathbf{d}\tilde{\mathbf{d}}\boldsymbol{\omega})^{n-1} \, (\mathbf{d}\tilde{\mathbf{d}}\tilde{\boldsymbol{\omega}})^{n-1} \; .$$

For $p + q =$ odd, it vanishes (unless $n = 1$) due to the grading.

Its symmetry depends on the values of p and q. The possibilities are:

$$\delta\boldsymbol{\omega}^{(p,q)} = \begin{cases} \mathbf{d}\boldsymbol{\lambda}^{(p-1,q)} + \tilde{\mathbf{d}}\boldsymbol{\lambda}'^{(p,q-1)} + b_{i_0 i_1 \dots i_{p+q}} x^{i_0} \theta^{i_1} \cdots \theta^{i_p} \chi^{i_{p+1}} \cdots \chi^{i_{p+q}} & (p, q > 0) \\ \mathbf{d}\boldsymbol{\lambda}^{(p-1,0)} + b_{i_0 i_1 \dots i_p} x^{i_0} \theta^{i_1} \cdots \theta^{i_p} & (p > 0, q = 0) \\ \tilde{\mathbf{d}}\boldsymbol{\lambda}'^{(0,q-1)} + b_{i_0 i_1 \dots i_q} x^{i_0} \chi^{i_1} \cdots \chi^{i_q} & (p = 0, q > 0) \\ c + b_i x^i & (p = q = 0) \end{cases}$$

with b fully antisymmetric (and constant).

N.B.: For $p, q > 0$, the last term does not survive irreducibility.

Easily generalized for towers of fields and up-to-second-order...

Special cases with enhanced structure

Recall: Scalar (0,0) led to more possibilities (odd number of fields) than p-form (p,0)

Similarly: A special mixed-symmetry field (p,p) allows more terms than a generic (p, q):

$$\mathcal{L}_{n+1}[\omega^{(p,p)}] = \frac{1}{(D-k)!} \int \mathrm{d}^D\theta \, \mathrm{d}^D\chi \, \boldsymbol{\eta}^{D-k} \mathbf{d}\boldsymbol{\omega} \, \widetilde{\mathbf{d}}\boldsymbol{\omega} \, (\mathbf{d}\widetilde{\mathbf{d}}\boldsymbol{\omega})^{n-1} \,, \quad k = (p+1)n + p \,.$$

This is not so surprising. After all, $p = 1$ is the graviton, and it works in all dimensions.

(1,1) Galileon and Linearized Gravity

In four dimensions, the Galileon for $h = \omega^{(1,1)}$ is identical to linearized Einstein-Hilbert:

$$S_{\text{LEH}}[h] = -\frac{1}{2}\int \mathrm{d}^4x\; h^{ij}\left(R_{ij} - \tfrac{1}{2}\eta_{ij}R\right) = -\frac{1}{4}\int \mathrm{d}^4x \int \mathrm{d}^4\theta\, \mathrm{d}^4\chi\, \boldsymbol{\eta}\, \boldsymbol{h}\, \mathbf{d}\tilde{\mathbf{d}}\boldsymbol{h}\,,$$

where $R_{ij} = \frac{1}{2}\left(\partial_i\partial_k h^k{}_j + \partial_k\partial_j h^k{}_i - \partial_i\partial_j h - \partial^2 h_{ij}\right),\; R = \eta^{ij}R_{ij}$.

The gauge transformations become identical to linearized diffeomorphisms:

$$\delta\boldsymbol{h} = \mathbf{d}\boldsymbol{\lambda}^{(0,1)} + \tilde{\mathbf{d}}\boldsymbol{\lambda}^{(1,0)}\,.$$

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In $D \geq 2n+1$ dimensions, the (1,1)-Galileon is linearized Lovelock at n-th order:

$$S_n^{\text{LL}}[h] \;=\; -\frac{1}{4}\frac{1}{(D-2n-1)!}\int \mathrm{d}^Dx \int \mathrm{d}^D\theta\, \mathrm{d}^D\chi\, \boldsymbol{\eta}^{D-2n-1}\, \boldsymbol{h}\,(\mathbf{d}\tilde{\mathbf{d}}\boldsymbol{h})^n\,.$$

Recall that Lovelock is a sum over dimensionally extended Euler densities:

$$S_{\text{Lovelock}} = \int d^Dx \sum_{n=0}^{\lfloor\frac{D-1}{2}\rfloor} \alpha_n \mathcal{L}_n\,, \quad \mathcal{L}_n = \frac{\sqrt{-g}}{2^n}\,\delta^{k_1 l_1 \ldots k_n l_n}_{i_1 j_1 \ldots i_n j_n} \prod_{r=1}^{n} R^{i_r j_r}{}_{k_r l_r}\,.$$

Covariantization

Scalar and *p*-form Galileons can be extended non-trivially to curved spacetime

Deffayet, Esposito-Farese, Vikman '09

- Promote partial derivatives to covariant
- Identify higher-derivative contributions on the field and the metric
- Introduce compensator terms to cancel 3 and 4 derivatives

Can we covariantize tensor Galileons?

Caution

- No-go theorem for interacting massless gravitons (at 2-derivative level)
  Boulanger, Damour, Gualtieri, Henneaux '00
- Unlike scalars and p-forms, where $\nabla_i = \partial_i$, for mixed-symmetry tensors $\nabla_i \neq \partial_i$
  - Additional complications, more higher-derivative terms
- Success (2-derivative field equations) does not guarantee consistency
  Aragone, Deser '80

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Just do it

The Gauss-Bonnet case

Defining

$$\boldsymbol{\nabla} = \theta^i \nabla_i , \qquad \widetilde{\boldsymbol{\nabla}} = \chi^i \nabla_i , \qquad \boldsymbol{Riem} = R_{ijkl}\theta^i\theta^j\chi^k\chi^l ,$$
$$\widetilde{\boldsymbol{h}}_l = h_{li}\theta^i , \qquad \boldsymbol{H}_l = H_{lij}\theta^i\chi^j , \qquad H_{lij} = \tfrac{3}{4}\nabla_l h_{ij} - \nabla_{(i}h_{j)l} ,$$

where $\nabla = \nabla^g$, the following action has 2nd order EOMs w.r.t. both g and h:

$$S_3[h,g] = S_{\text{LL}}[g] + \int \mathrm{d}^5x \int \mathrm{d}^5\theta\, \mathrm{d}^5\chi\, \sqrt{-g}\, \left(\boldsymbol{\nabla} \boldsymbol{h}\, \widetilde{\boldsymbol{\nabla}} \boldsymbol{h}\, \boldsymbol{\nabla} \widetilde{\boldsymbol{\nabla}} \boldsymbol{h} + \widetilde{\boldsymbol{\nabla}} \boldsymbol{h}\, \widetilde{\boldsymbol{h}}_l\, \boldsymbol{H}^l\, \boldsymbol{Riem} \right) .$$

Epilogue

Take-home messages

- Galileon-type Lagrangians have a beautiful structure and physical applications
- We suggested a natural and universal formulation in terms of graded variables ...
- ... which reveals a further generalization to mixed-symmetry tensor fields ...
- ... by-producing an elegant formula for linearized Lovelock in any dimension ...
- ... and a highly non-trivial covariantization for linearized 5d Gauss-Bonnet

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`thanks`