Entanglement, Holography and Causal Diamonds

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based on **1509.00113** with de Boer, Myers and Neiman
and **1606.03307** with de Boer, Haehl and Myers
[see also 1604.03110 by other authors]

Alexander von Humboldt
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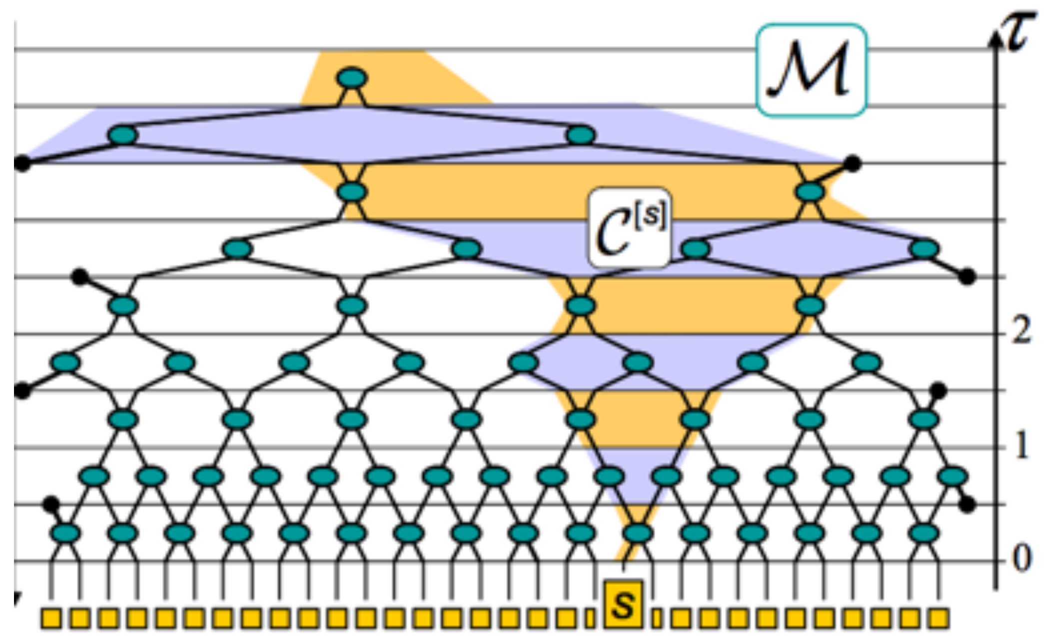
Motivation

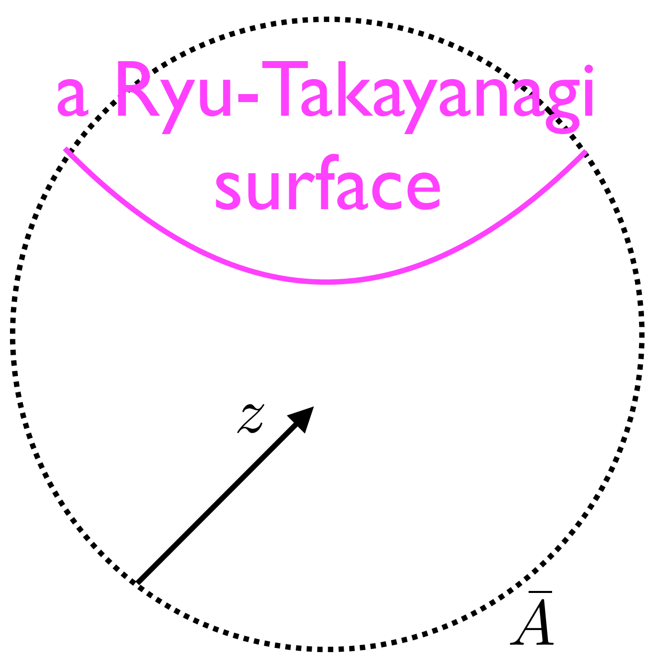
Motivation

real space RG (MERA):



physics of subregions in quantum-many body systems, QFTs and holography

bulk const-t slice



$$S_{EE} = -\mathrm{tr}\,\rho_A \log \rho_A \equiv \frac{\text{Area}}{4G_N}$$

where $\mathcal{H}_{QG} \equiv \mathcal{H}_{hQFT} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$ & $\rho_A = \mathrm{tr}_{\bar{A}}\rho$

Questions behind 1509.00113 and 1606.03307:

1) can a subregion decomposition in a QFT be understood geometrically?

2) what would be then the relation to holography?

Setup

Setup

any conformal field theory (CFT) in d spacetime dimensions

$+$

spatial subregions $=$ spheres on some constant time slice

$+$

at least initially, $\rho = |0\rangle\langle 0| + \epsilon\, \delta\rho$ with $\epsilon \ll 1$

spherical regions in CFTs & de Sitter

1509.00113 with de Boer, Myers and Neiman

Entanglement first law in CFTs I

Consider small perturbation of some reference density matrix $\rho = \rho_0 + \delta\rho$

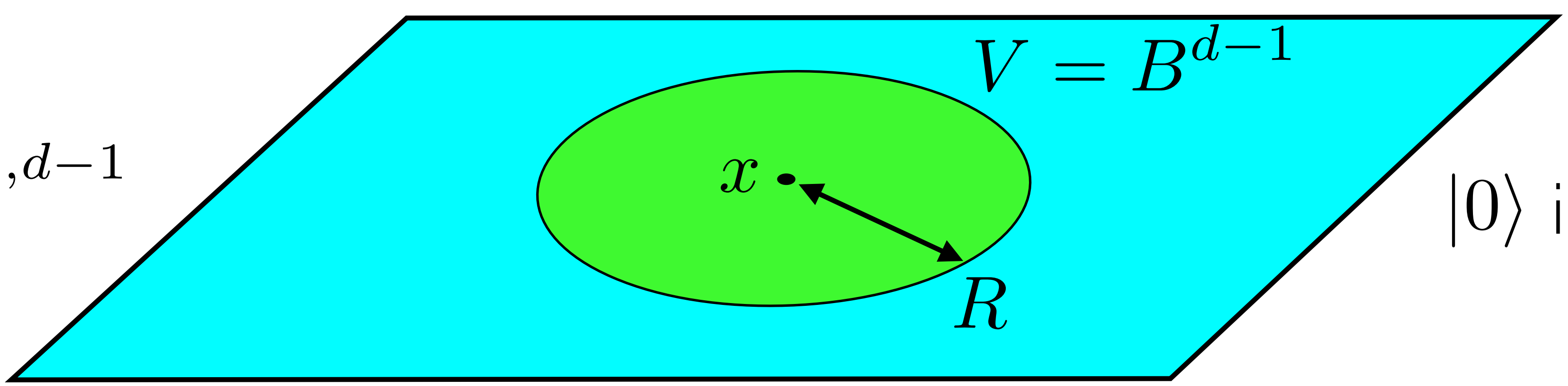
The change in the entropy is equal to the change in <the modular Hamiltonian>

$$\delta S = -\mathrm{tr}\,(\rho \log \rho) - S_0 = \delta \langle H_{mod} \rangle$$

In general, we expect $H_{mod} \equiv \log \rho_0$ to be nonlocal, but for $\rho_0 = \mathrm{tr}_V |0\rangle\langle 0|$:

$$H_{mod} = c' + 2\pi \int_{|\vec{x}-\vec{x}'|^2 \leq R^2} d^{d-1}x' \, \frac{R^2 - |\vec{x} - \vec{x}'|^2}{2R} T_{tt}(x')$$

$t = 0$ of $\mathbb{R}^{1,d-1}$



$|0\rangle$ is the vacuum$_{\mathrm{CFTd}}$

Casini, Huerta & Myers 1102.0440

Entanglement first law in CFTs 2

As a result, the change in the entanglement entropy for small perturbations of $|0\rangle$ is

$$\delta S_B = 2\pi \int_{|\vec{x}-\vec{x}'|^2 \leq R^2} d^{d-1}x' \boxed{\frac{R^2 - |\vec{x}-\vec{x}'|^2}{2R}} \langle T_{tt}\rangle(x')$$

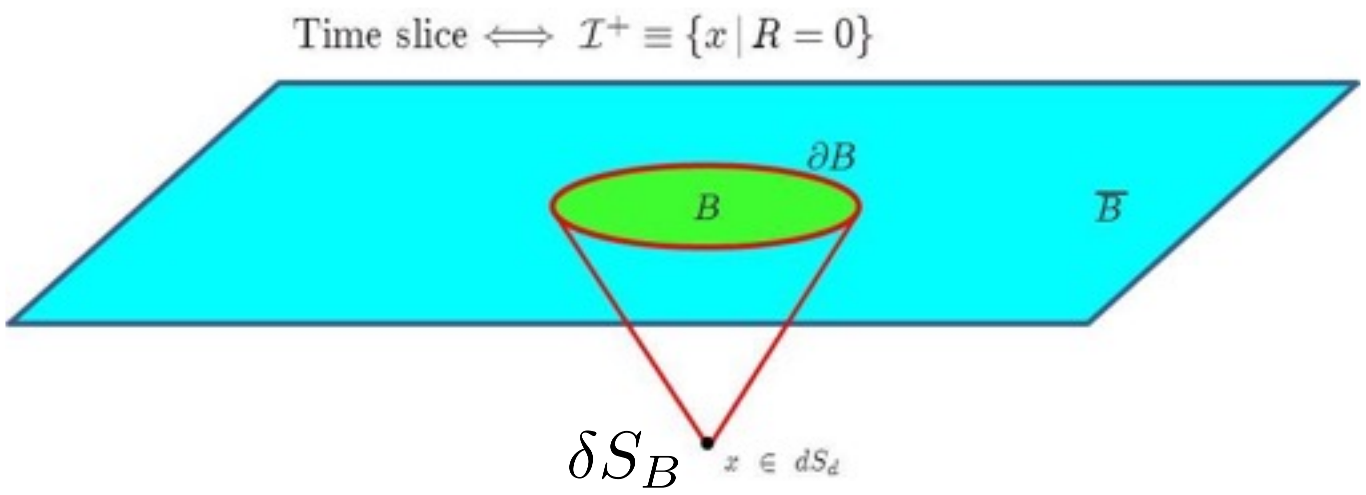
This equation is the main player of my talk

Key insights of 1509.00113

→ δS obeys a wave equation on de Sitter

→ de Sitter appears as a moduli space of spheres

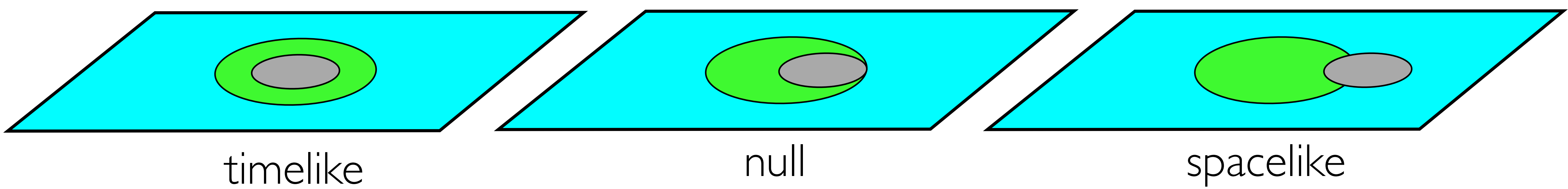
Relation to de Sitter geometry



$$\nabla_a \nabla^a \big|_{dS_d} \delta S_B - m^2 \delta S_B = 0$$

with $\quad m^2 L^2 = -d$

Causal relations between points in dS_d $\Leftrightarrow$ partial order between B's on t=0:



the subgroup of SO(2,d) preserving a given time slice

dS_d = SO(1,d) / SO(1,d-1)

the subgroup of SO(2,d) preserving a sphere on a time slice

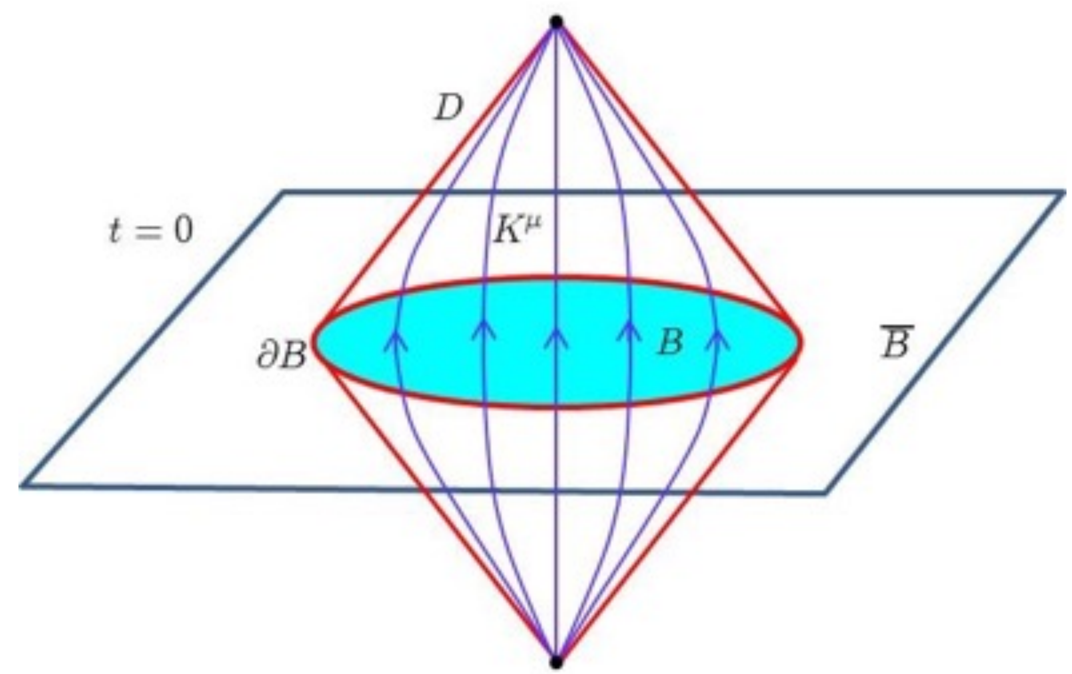
Moduli space of causal diamonds and associated observables in CFTs

1606.03307 with de Boer, Haehl and Myers

From $T_{\mu\nu}$ to $\mathcal{O}_{\mu_1 \ldots \mu_l}$

We can write $\delta S_B = 2\pi \int_{|\vec{x}-\vec{x}'|^2 \leq R^2} d^{d-1}x' \, \frac{R^2 - |\vec{x}-\vec{x}'|^2}{2R} \langle T_{tt} \rangle (x') \sim \int_\diamond d^d\xi \, |K|^{-2} \, K^\mu K^\nu \, \langle T_{\mu\nu}(\xi) \rangle$

which suggests to think about δS_B as a natural causal diamond observable associated with $T_{\mu\nu}$



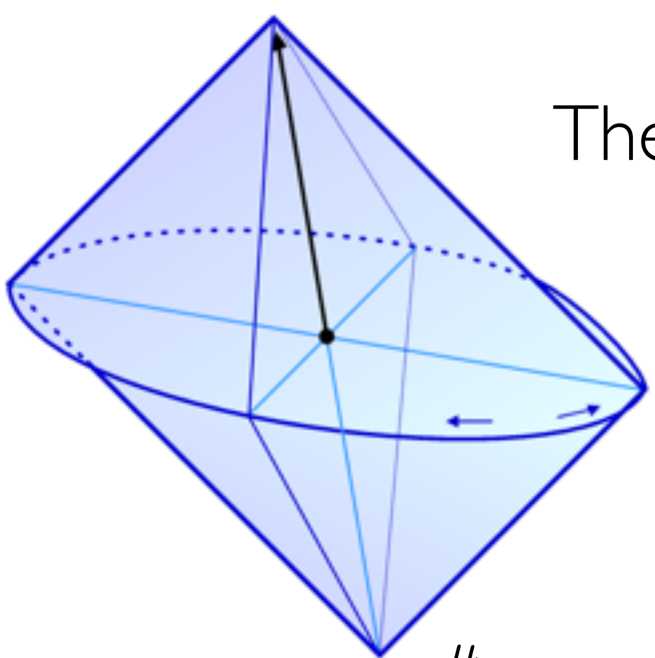
How about other primaries in a CFT? We propose the following definition

$$Q(\mathcal{O}, x, y) \sim \int_\diamond d^d\xi \, |K|^{\Delta - l - d} K^{\mu_1} \cdots K^{\mu_l} \langle O_{\mu_1 \ldots \mu_l}(\xi) \rangle$$

This quantity has a holographic interpretation, but in general it does not "live" in dS_d

Moduli space of causal diamonds

General spherical surface on some constant-time slice is specified by the coordinates of the tips of the corresponding causal diamond: x^μ & y^μ :



There is a unique SO(2,d)-invariant metric parametrized by x^μ & y^μ:

$$-\frac{4\,L^2}{(x-y)^2}\left(\eta_{\mu\nu}+\frac{2(x_\mu-y_\mu)(x_\nu-y_\nu)}{-(x-y)^2}\right)dx^\mu dy^\nu$$

Its signature is (d,d) and arises as SO(2,d) / [SO(1,d-1)xSO(1,1)]

$Q(\mathcal{O},x,y)\sim\int_\diamond d^d\xi\,|K|^{\Delta-l-d}K^{\mu_1}\cdots K^{\mu_l}\langle O_{\mu_1\ldots\mu_l}(\xi)\rangle$ obey now a set of intricate local EOMs

dS_d is a particular submanifold of this much larger moduli space

Holographic interpretation

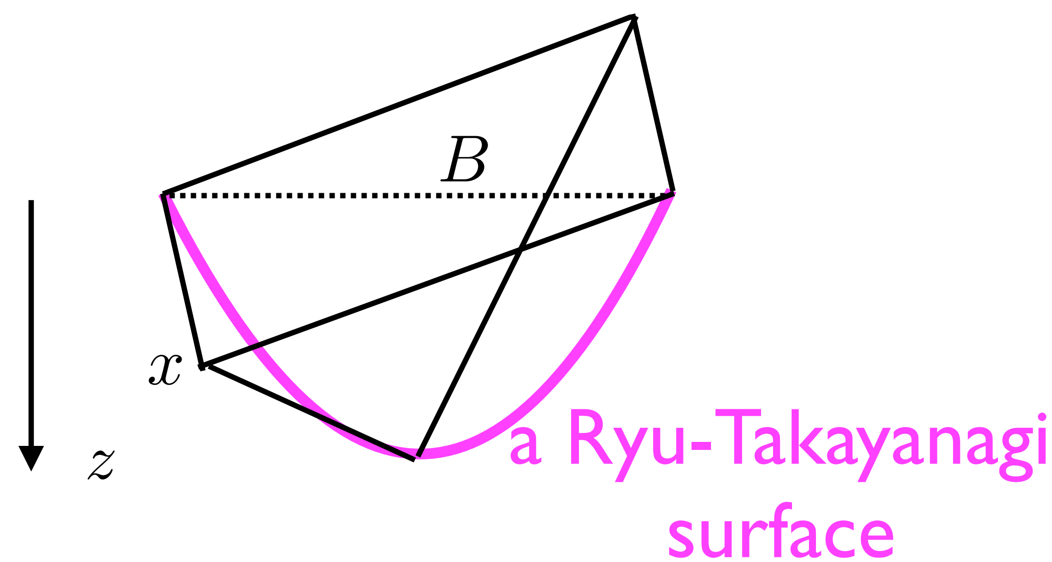
1606.03307 with de Boer, Haehl and Myers

RT-surfaces averages of bulk fields

In holographic theories for scalar operator $\mathcal{O}$ dual to free bulk field ϕ we have

$$\nabla^2_{AdS_{d+1}} \phi - m^2 \phi = 0$$

with $\phi = \langle \mathcal{O} \rangle \cdot z^{\Delta} + \dots$



$$Q(\mathcal{O}, x, y) \sim \int_{\text{a Ryu-Takayanagi surface}} d^{d-1}\sigma \sqrt{g_{\text{induced}}}\, \phi$$

Can be proven/demonstrated in many different ways: group theory/ explicit sols for ϕ

This provides a completely new set of nonlocal observables in holography.

Summary

1509.00113 with de Boer, Myers and Neiman

1606.03307 with de Boer, Haehl and Myers

Summary

$$-\frac{4L^2}{(x-y)^2}\left(\eta_{\mu\nu}+\frac{2(x_\mu-y_\mu)(x_\nu-y_\nu)}{-(x-y)^2}\right)dx^\mu dy^\nu$$

$$\downarrow$$

Geometrization of causal diamonds in CFTs:

$$\frac{L^2}{R^2}\left(-dR^2+d\vec{x}^2\right)$$

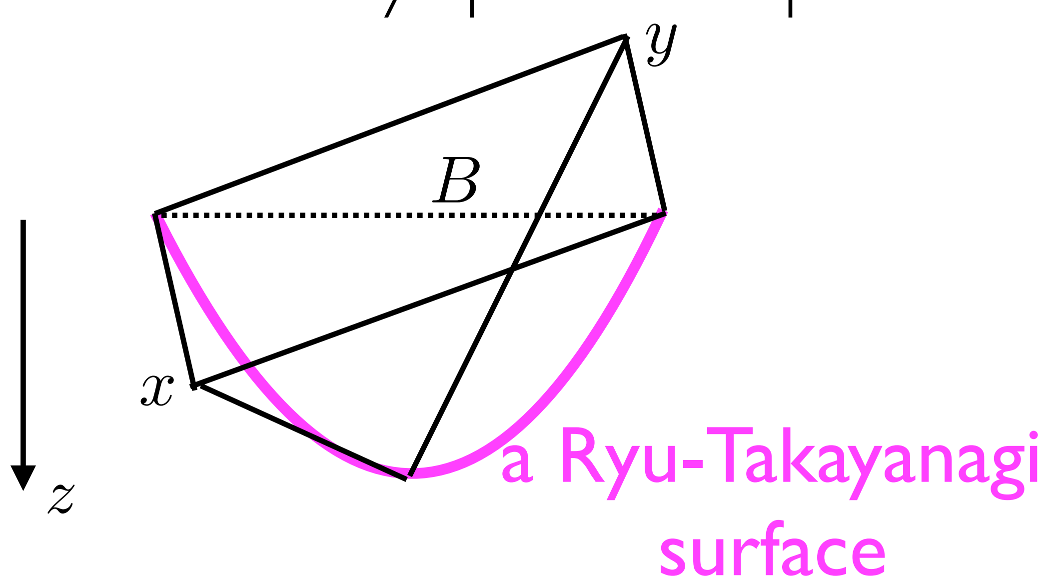
Natural (conformally-invariant) quantities associated with primary in such region are

$$Q(\mathcal{O},x,y)\sim\int_\diamond d^d\xi\,|K|^{\Delta-l-d}K^{\mu_1}\cdots K^{\mu_l}\langle O_{\mu_1\ldots\mu_l}(\xi)\rangle$$

Holography: RT surfaces anchored on bdry spheres as probes of bulk matter configs

$$\nabla^2_{AdS_{d+1}}\phi-m^2\,\phi=0$$

with $\phi=\langle\mathcal{O}\rangle\cdot z^\Delta+\ldots$



$$Q(\mathcal{O},x,y)$$
$$\sim$$
$$\int_{\text{a Ryu-Takayanagi surface}} d^{d-1}\sigma\,\sqrt{g_{\text{induced}}}\,\phi$$